Transiogram – A spatial correlation measure for categorical data

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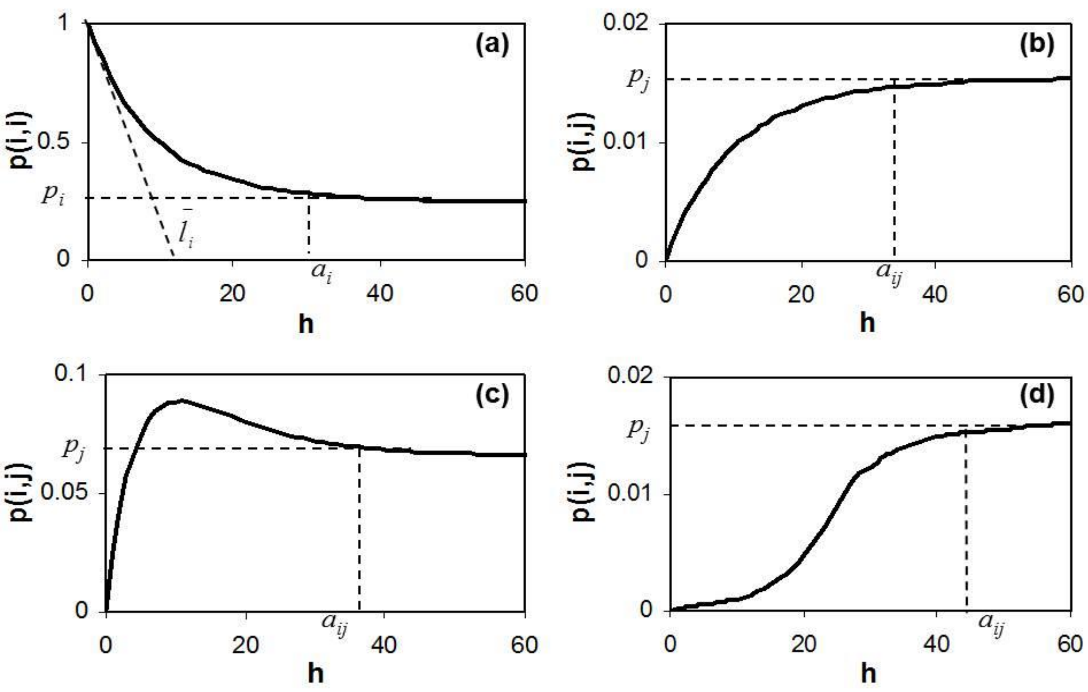


First law of geography

- Spatial correlation is the basic property of geographic data, which account for the majority of spatial data.
- Think about the first law of geography – Near things are more related than distant things.
- Spatial correlation analysis is the basic approach for analyzing spatial patterns.
- Spatial data include data of continuous spatial variables and data of categorical spatial variables.

Categorical spatial data have spatial auto-correlation and cross correlation

- A categorical variable may include several classes.
- Each class has auto-correlation.
- Each pair of classes has cross-correlation.
- All spatial correlations may have anisotropy.
- Class spatial interdependcy/cross-correlation may have spatial asymmetry.

Transition probability matrix

$$
\begin{array}{c} \\ \text{Rain} \\ \text{Nice} \\ \text{Snow} \end{array}
\begin{array}{c}
\begin{array}{ccc} \text{Rain} & \text{Nice} & \text{Snow} \end{array} \\
\begin{pmatrix} .500 & .250 & .250 \\ .500 & .000 & .500 \\ .250 & .250 & .500 \end{pmatrix}
\end{array}
$$

which describes a stationary Markov chain.

Spatial transition probability matrix

Soil type	1	2	3	4	5	6	7
			TPM in the x-direction				
1	.838	.030	.055	.010	.046	.012	.009
2	.202	.614	.019	.013	.133	.006	.013
3	.098	.004	.804	.009	.022	.052	.011
4	.033	.000	.041	.633	.131	.077	.086
5	.050	.010	.002	.014	.849	.049	.026
6	.029	.000	.051	.025	.083	.732	.080
7	.040	.000	.004	.055	.139	.090	.672

which describes a spatially-stationary Markov chain.

Transition probability matrix

TPM is widely used to represent class dependencies. But it reveals limited information of spatial class relationships

Transition-Probability Matrix

- Example
  - *system with three states*

$$\Pi \equiv \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.9 & 0.1 & 0.0 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}$$

If in state 1, will stay in state 1 with probability 0.1

If in state 1, will move to state 3 with probability 0.4

Never go to state 3 from state 2

- Requirements of transition-probability matrix
  - *all probabilities non-negative, and no greater than unity*
  - *sum of each row is unity*
  - *probability of staying in present state may be non-zero*

How about multiple spatial steps or a long lag distance h?

Transiograms

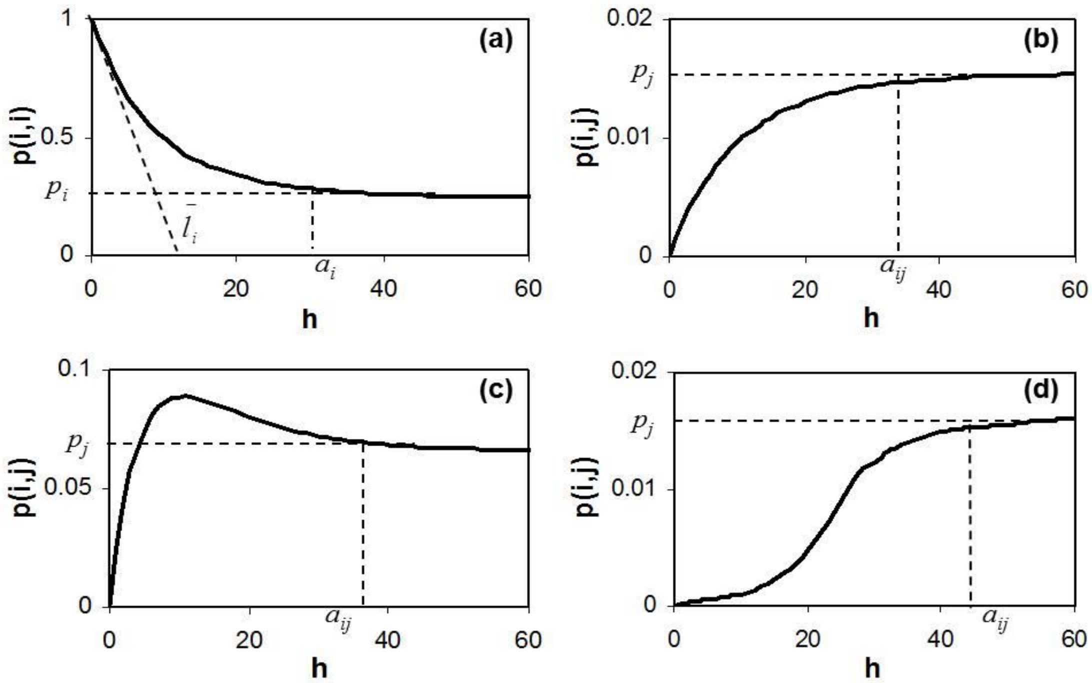
$$p_{ij}(\mathbf{h}) = \Pr[Z(\mathbf{u}+\mathbf{h}) = j \mid Z(\mathbf{u}) = i]$$

1. A transiogram is theoretically defined as a two-point transition probability function over the separation distance (i.e., spatial lag)

2. Graphically a transiogram $p_{ij}(\mathbf{h})$ is a transition probability curve (or diagram) with increasing lag **h** from zero to a certain distance

3. Transition probability vs spatial lag curves appeared in publications as early as 1969 (Schwarzacher, 1969).

Typical transiogram shapes



- Typical features and indices: sill, correlation range, curve shape, peak (and trough), peak height ratio.

- Cross transiograms are asymmetric and can be unidirectional, and they have tail and head classes.

Different types of Transiograms

We have

1. experimental (or empirical or sample) transiograms – estimated from sample data.

2. theoretical transiograms or transiogram models – fitted mathematical models

3. idealized transiograms – smooth curves calculated from transition probability matrix.

4. exhaustive transiograms – a kind of experimental transiograms directly estimated from exhaustive map data

Idealized transiograms

Directly computed from a TPM

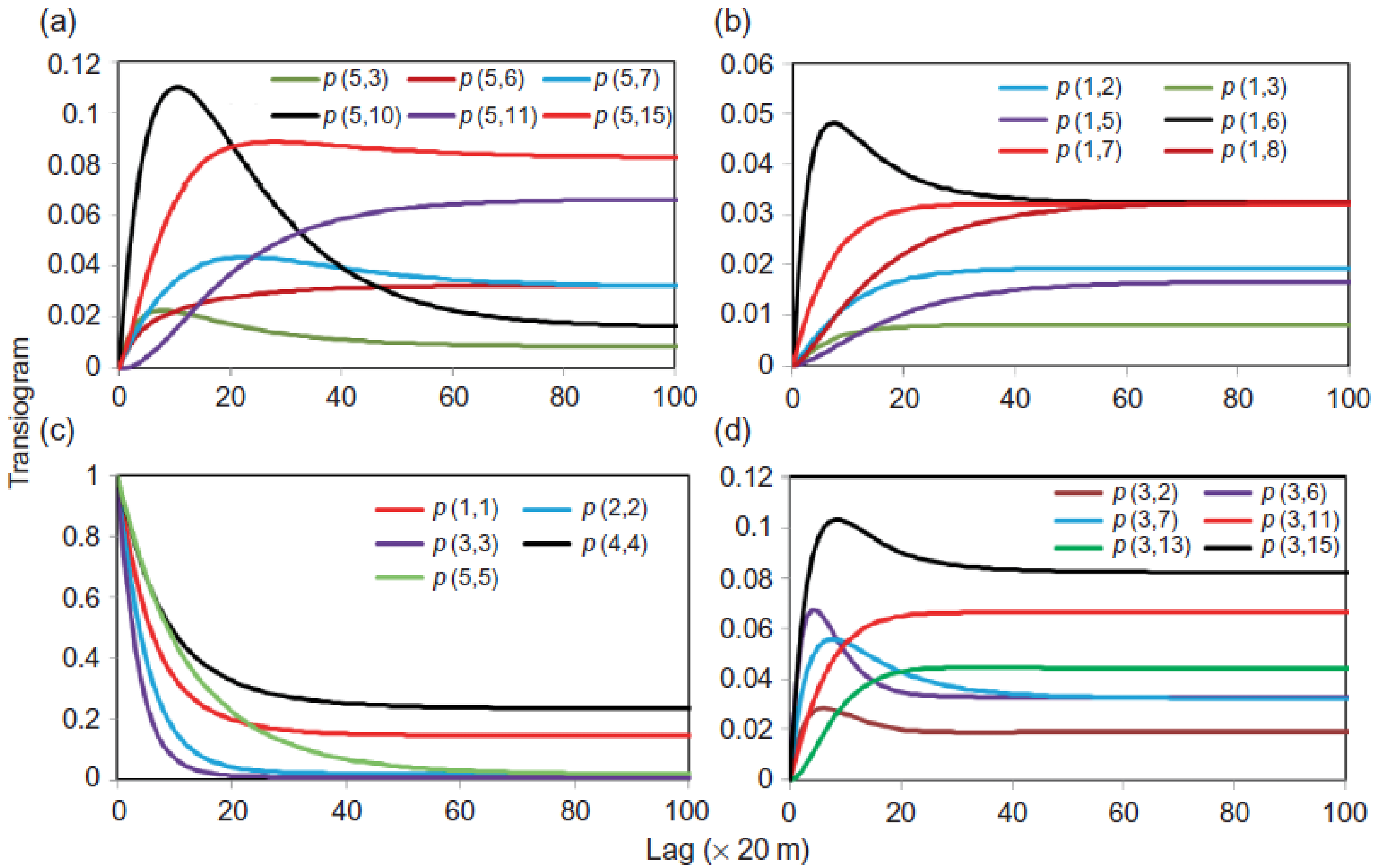


Figure 4. Some idealized transiograms calculated from a unidirectional TPM which is computed from soil map data.

Estimation of idealized transiograms

$$\boldsymbol{P}(n) = [p_{ij}(n)] = [\boldsymbol{P}(1)]^n = [p_{ij}(1)]^n$$

- Simply by self-multiplication.

- A *n*-step TPM is equal to a *n*-powered one-step TPM. (step --- a pixel length or a time period).

- Download computer program: **Idealized transiograms from TPM** http://gis.geog.uconn.edu/weidong/MCG/MCG_Software.htm

Experimental transiograms

Directly estimated from sample data

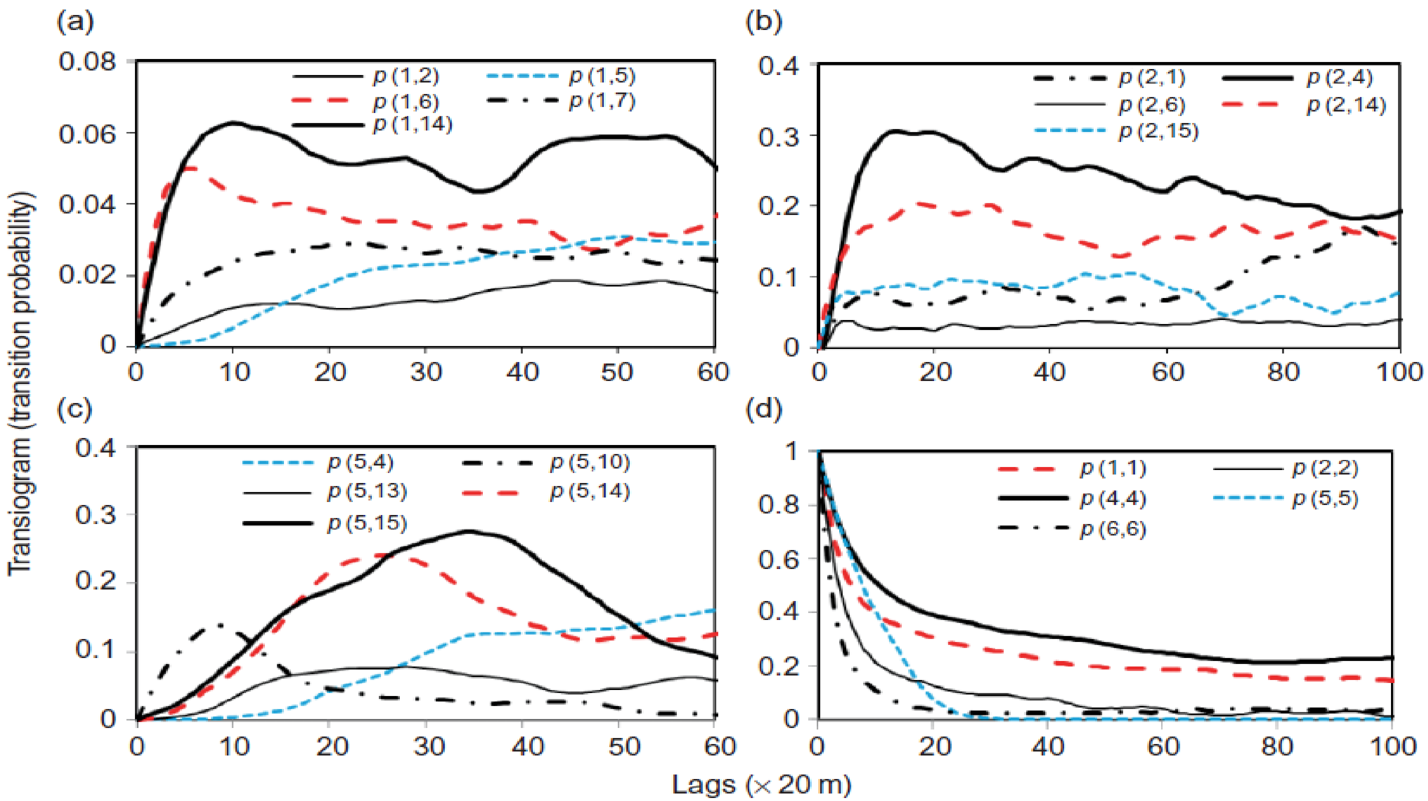


Figure 1. Unidirectional experimental transiograms estimated from a soil map data set along the east-to-west direction. Pixel size is 20 m × 20 m. Here $p(i, j)$ refers to the experimental transiogram $\hat{p}_{ij}(\mathbf{h})$.

Estimation of experimental transiograms

$$\hat{p}_{ij}(\mathbf{h}) = \frac{F_{ij}(\mathbf{h})}{\sum_{k=1}^{n} F_{ik}(\mathbf{h})}$$

Count the frequencies of spatial transitions

- May be estimated uni-directionally, bi-directionally, multi-directionally, or omni-directionally. And may consider anisotropy.
- Similar to estimating TPM, but need to consider a series of different lag values.
- May need to consider a tolerance width (i.e., h = h±Δh/2)
- May need to consider a tolerance angle or all directions.
- Download computer program: **Omni-directional experimental transiogram estimation** http://gis.geog.uconn.edu/weidong/MCG/MCG_Software.htm

Discrepancy between idealized transiograms and experimental transiograms

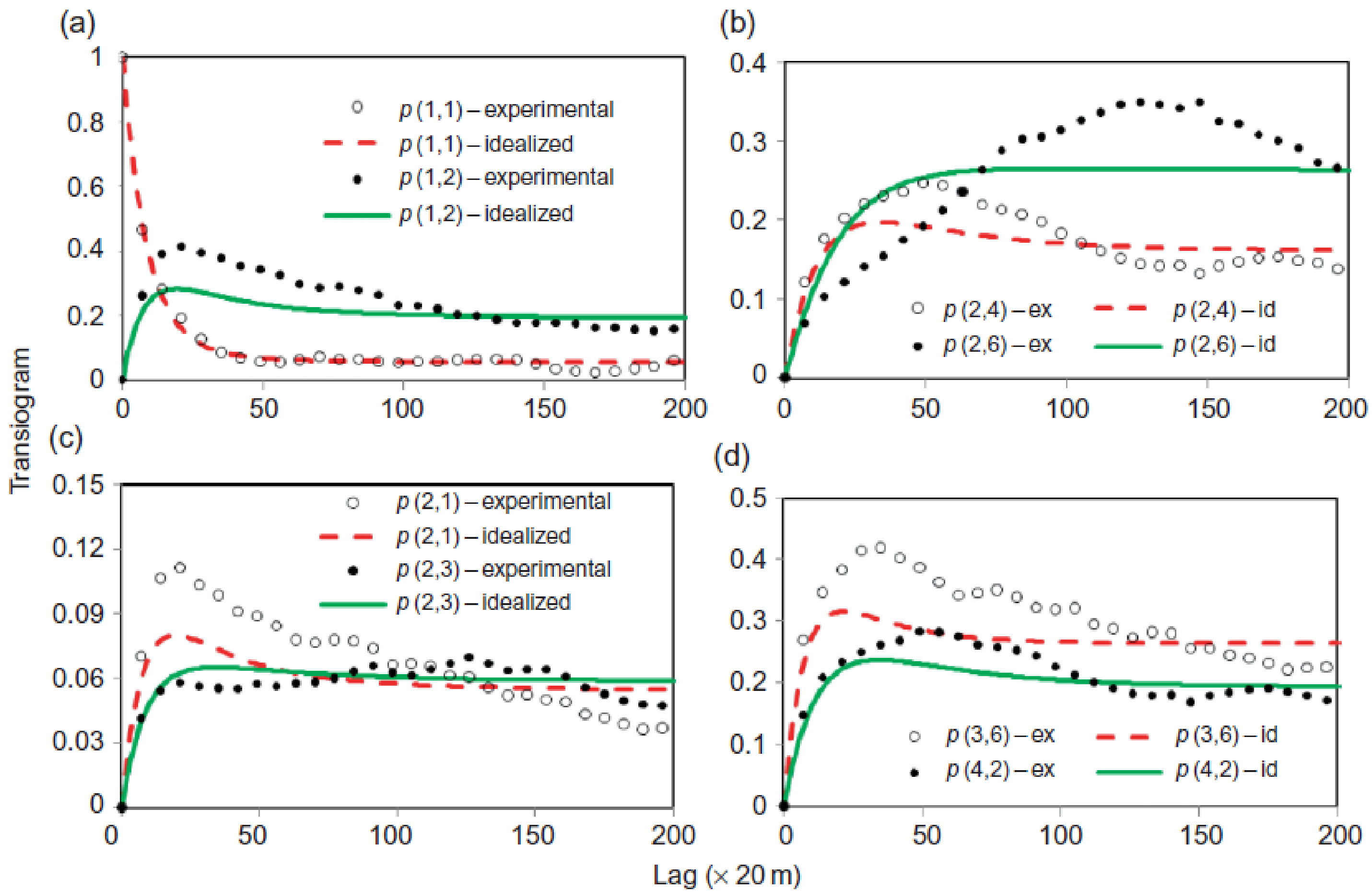
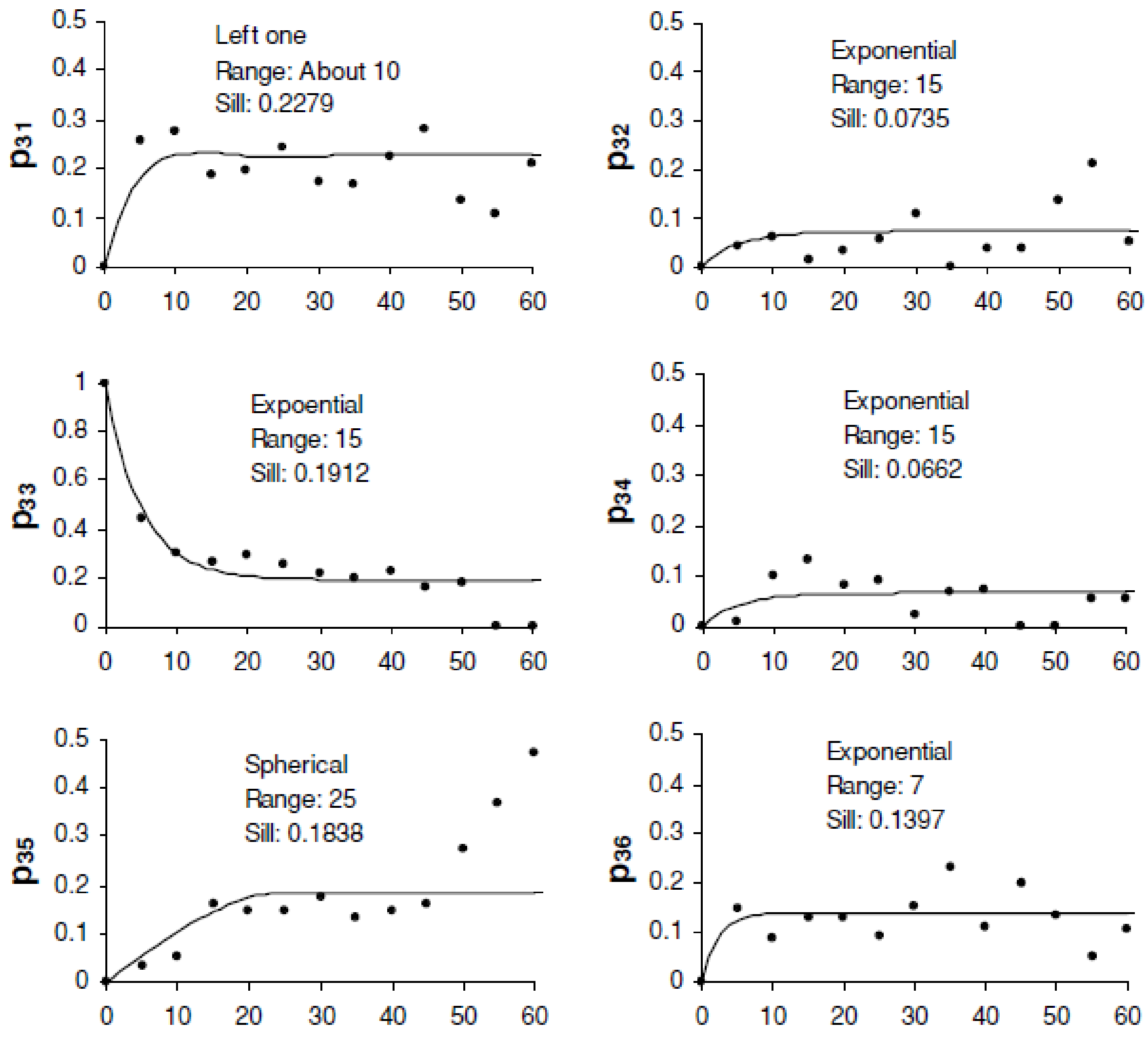


Figure 6. Idealized transiograms and their corresponding experimental transiograms calculated from land cover data.

Transiogram models

Mathematical models fitted to experimental transiograms



Transiogram modeling

- Mainly provide transition probability values at any lag values to related spatial models/simulation algorithms (e.g., MCRF sequential class simulation algorithm) for spatial simulation of categorical fields.

- Represent a transiogram or describe a class auto-correlation/cross-correlation with a few model parameters.

- Download computer program: (1) **TGRAM** or (2) **Basic math models for transiogram modeling**.
  http://gis.geog.uconn.edu/weidong/MCG/MCG_Software.htm

Methods and models for transiogram joint modeling

Table 1
Fitting models of transiograms supported by the proposed framework.

Method	Model name	Equation[a]	Input parameters or requirements[b]
Mathematical model	*For auto-transiograms*		
	Linear	$p_{ii}(\mathbf{h}) = \begin{cases} 1-(1-p_i)\mathbf{h}/a_i, & \mathbf{h} < a_i \\ p_i, & \mathbf{h} \geq a_i \end{cases}$	a_i
	Spherical	$p_{ii}(\mathbf{h}) = \begin{cases} 1-(1-p_i)[1.5(\mathbf{h}/a_i)-0.5(\mathbf{h}/a_i)^3], & \mathbf{h} < a_i \\ p_i, & \mathbf{h} \geq a_i \end{cases}$	a_i
	Exponential	$p_{ii}(\mathbf{h}) = 1-(1-p_i)[1-\exp(-3\mathbf{h}/a_i)]$	a_i
	Gaussian	$p_{ii}(\mathbf{h}) = 1-(1-p_i)\{1-\exp[-3(\mathbf{h}/a_i)^2]\}$	a_i
	Cosine-exponential	$p_{ii}(\mathbf{h}) = 1-(1-p_i)[1-\exp(-3\mathbf{h}/a_i)\cos(2\pi\mathbf{h}/\lambda)]$	a_i, λ
	Cosine-Gaussian	$p_{ii}(\mathbf{h}) = 1-(1-p_i)\{1-\exp[-3(\mathbf{h}/a_i)^2]\cos(2\pi\mathbf{h}/\lambda)\}$	a_i, λ
	For cross-transiograms ($i \neq j$)		
	Linear	$p_{ij}(\mathbf{h}) = \begin{cases} p_j\mathbf{h}/a_{ij}, & \mathbf{h} < a_{ij} \\ p_j, & \mathbf{h} \geq a_{ij} \end{cases}$	a_{ij}
	Spherical	$p_{ij}(\mathbf{h}) = \begin{cases} p_j[1.5(\mathbf{h}/a_{ij})-0.5(\mathbf{h}/a_{ij})^3], & \mathbf{h} < a_{ij} \\ p_j, & \mathbf{h} \geq a_{ij} \end{cases}$	a_{ij}
	Exponential	$p_{ij}(\mathbf{h}) = p_j[1-\exp(-3\mathbf{h}/a_{ij})]$	a_{ij}
	Gaussian	$p_{ij}(\mathbf{h}) = p_j\{1-\exp[-3(\mathbf{h}/a_{ij})^2]\}$	a_{ij}
	Cosine-exponential	$p_{ij}(\mathbf{h}) = p_i[1-\exp(-3\mathbf{h}/a_{ij})\cos(2\pi\mathbf{h}/\lambda)]$	a_{ij}, λ
	Cosine-Gaussian	$p_{ij}(\mathbf{h}) = p_j\{1-\exp[-3(\mathbf{h}/a_{ij})^2]\cos(2\pi\mathbf{h}/\lambda)\}$	a_{ij}, λ
	Gamma-exponential[c]	$p_{ij}(\mathbf{h}) = p_j\left[1-\exp(-3\mathbf{h}/a_{ij})+\frac{w}{\Gamma(\alpha)\beta^\alpha}\left(\frac{\mathbf{h}}{a_{ij}}\right)^{\alpha-1}\exp\left(\frac{-\mathbf{h}}{\beta a_{ij}}\right)\right], \quad \alpha>1, \beta>0$	a_{ij}, α, β, w
	Gamma-Gaussian[c]	$p_{ij}(\mathbf{h}) = p_j\left[1-\exp(-3\mathbf{h}^2/a_{ij}^2)+\frac{w}{\Gamma(\alpha)\beta^\alpha}\left(\frac{\mathbf{h}}{a_{ij}}\right)^{\alpha-1}\exp\left(\frac{-\mathbf{h}}{\beta a_{ij}}\right)\right], \quad \alpha>1, \beta>0$	a_{ij}, α, β, w
	Gamma-spherical[c]	$p_{ij}(\mathbf{h}) = \begin{cases} p_j\left[1.5\frac{\mathbf{h}}{a_{ij}}-0.5\left(\frac{\mathbf{h}}{a_{ij}}\right)^3+\frac{w}{\Gamma(\alpha)\beta^\alpha}\left(\frac{\mathbf{h}}{a_{ij}}\right)^{\alpha-1}\exp\left(\frac{-\mathbf{h}}{\beta a_{ij}}\right)\right], & 0<\mathbf{h}<a_{ij} \\ p_j\left[1+\frac{w}{\Gamma(\alpha)\beta^\alpha}\left(\frac{\mathbf{h}}{a_{ij}}\right)^{\alpha-1}\exp\left(\frac{-\mathbf{h}}{\beta a_{ij}}\right)\right], & \mathbf{h} \geq a_{ij} \end{cases}, \alpha>1, \beta>0$	a_{ij}, α, β, w
	Infer p_{ij} from p_{ji}	$p_{ij}(\mathbf{h}) = \frac{p_j}{p_i}p_{ji}(\mathbf{h})$	After $p_{ji}(\mathrm{h})$ has been fitted
	For auto and cross-transiograms		
	Fit by (1.0 – Others)	$p_{ik}(\mathbf{h}) = 1-\sum_{j=1, j\neq k}^{n} p_{ij}(\mathbf{h})$	After all other $p_{ij}(\mathrm{h})$ in a transiogram matrix row have been fitted, the left one is fitted by this equation
Linear interpolation	Linear interpolation[d]	$p_{ij}(\mathbf{h}) = \frac{\hat{p}_{ij}(\mathbf{h}_k)(\mathbf{h}_{k+1}-\mathbf{h})+\hat{p}_{ij}(\mathbf{h}_{k+1})(\mathbf{h}-\mathbf{h}_k)}{\mathbf{h}_{k+1}-\mathbf{h}_k}$	Output pixel size (pSize), number of output lag values (lagNum)

[a] $\mathrm{h} \geq 0$ for all equations.

[b] a_i = auto-correlation range; a_{ij} = cross-correlation range; p_i = proportion of class i; λ is the wavelength of the cosine function; α is the shape parameter of the gamma distribution function; β is the scale parameter of the gamma distribution function; w is a weight parameter for the gamma distribution function component in the composite model of Gamma-exponential, Gamma-Gaussian and Gamma-spherical; In the Gamma distribution function, $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt$.

[c] Gamma-exponential, Gamma-Gaussian and Gamma-spherical model for cross-transiograms were provided in Li et al. (2012).

[d] Linear interpolation model was provided in Li and Zhang (2010a, 2010b). Other fitting models were provided in Li (2007a).

Independent uses

1. Transiograms may be used independently to describe the spatial patterns of a variety of spatial categories, for example, landscape classes.

2. Transiograms also can be used in time dimension to represent the time correlations of classes.

3. Continuous variables may be discretized into a series of grades based on some thresholds, and then characterized by transiograms.

4. Other potential uses.

References

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Current and Potential Applications of the MCRF approach with transiograms

1. Predictive soil mapping: mapping soil types or soil textural classes based om sample data, legacy data and other auxiliary data.

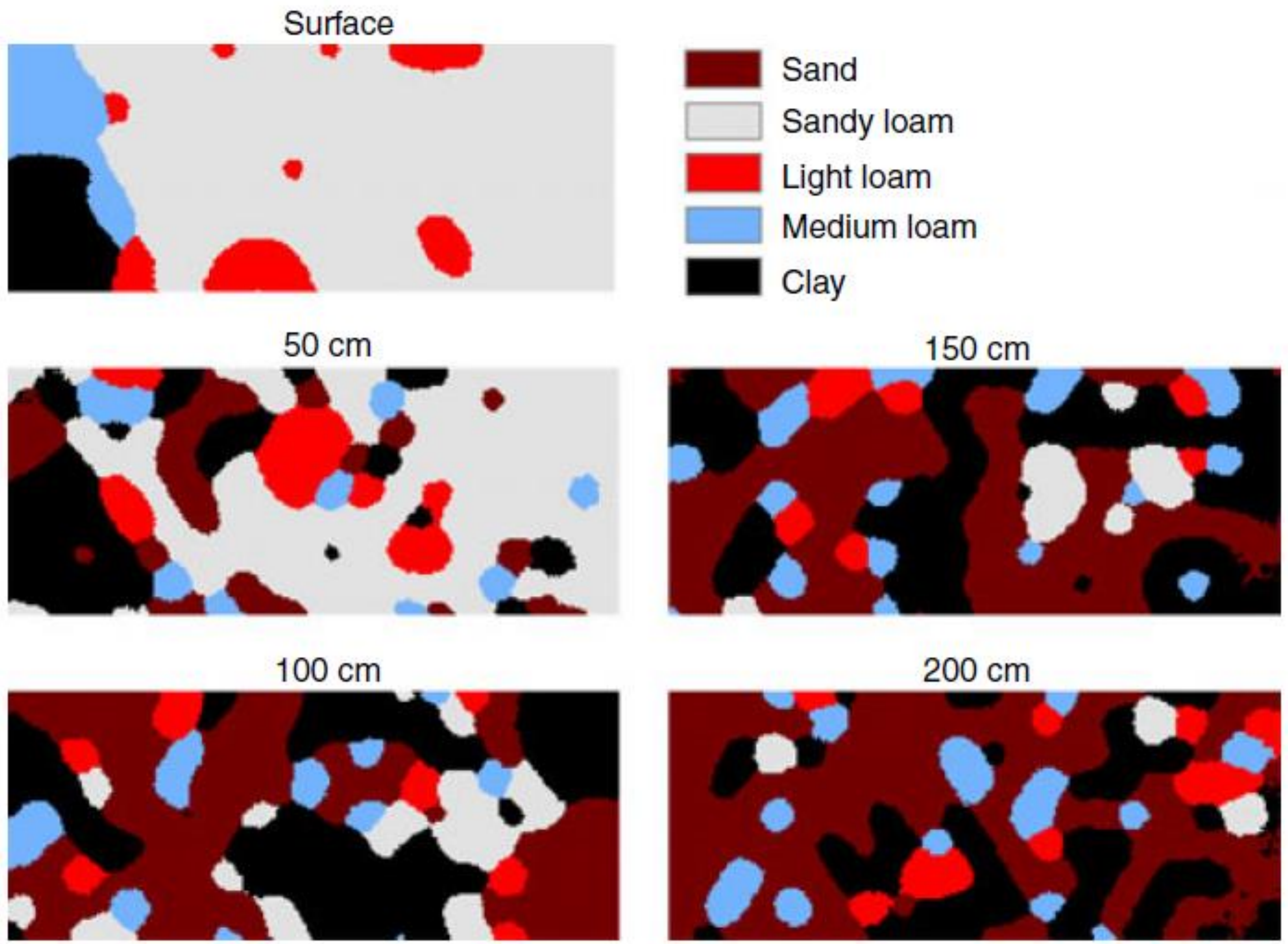
2. Mapping land cover/use classes and their changes in spatial or spatiotemporal dimensions.

3. Mapping subsurface facies such as lithologies in two to three dimensions.

4. Post-processing land cover classification maps from remotely sensed imagery to improve accuracy and reduce noise.

5. Indirectly used to detect taller urban buildings based on building shadow classification.

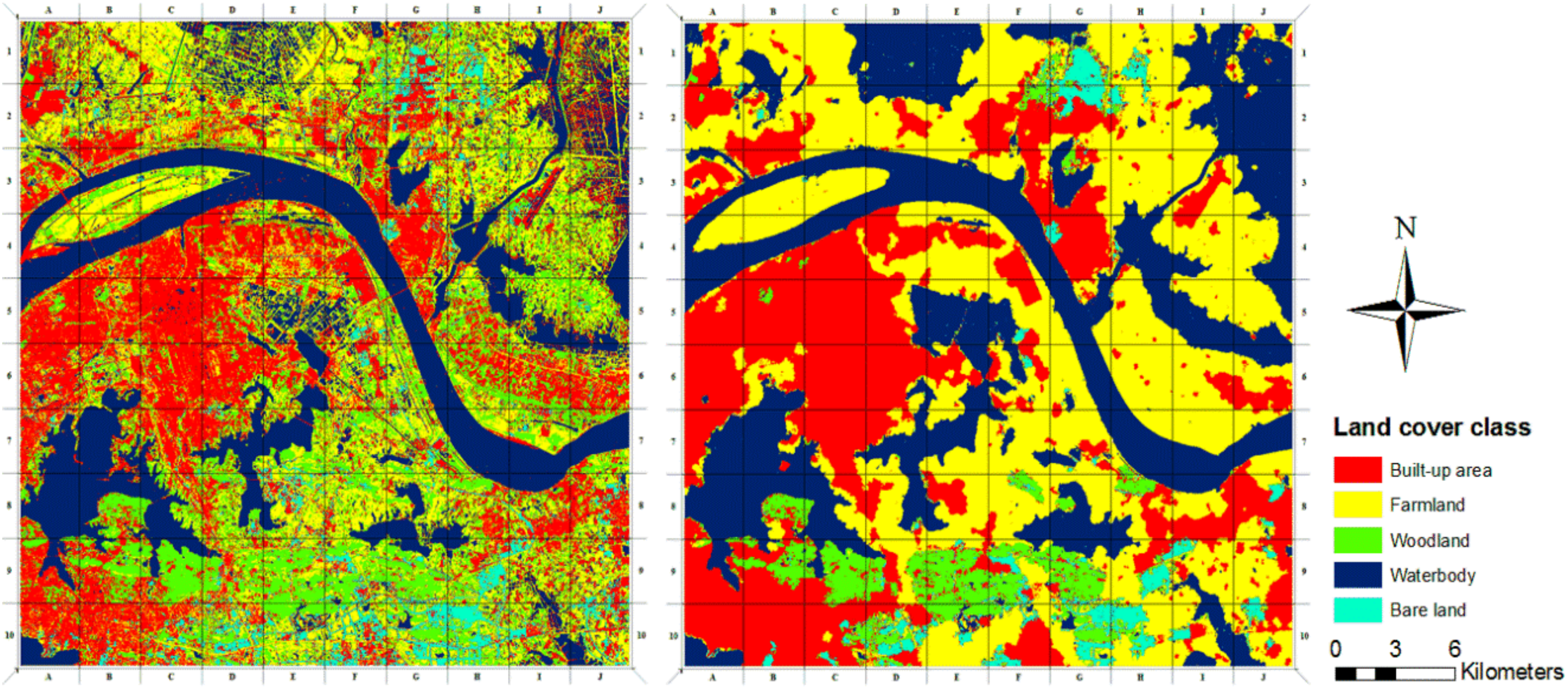
Soil textural mapping



Optimal prediction maps of soil textural classes at five depths, based on maximum occurrence probabilities estimated from 100 simulated realizations generated by the MCRF sequential class simulation algorithm

Land cover/use post-classification

based on expert-interpreted sample data and pre-classification



(*a*) (*b*)

Land cover classification from remotely sensed imagery: Neural Network pre-classification (**a**), and corresponding coMCRF post-classification (**b**).

Other applications

- Detection of mid-rise and taller buildings (MTBs) based on shadow classification post-processed by SS-coMCRF simulation

- Soil/land cover map updating by coMCRF simulation

- 3D subsurface facies (e.g., lithologies) modeling

- Spatiotemporal modeling of land cover/use

Detected mid-rise and taller buildings

In Guangzhou City

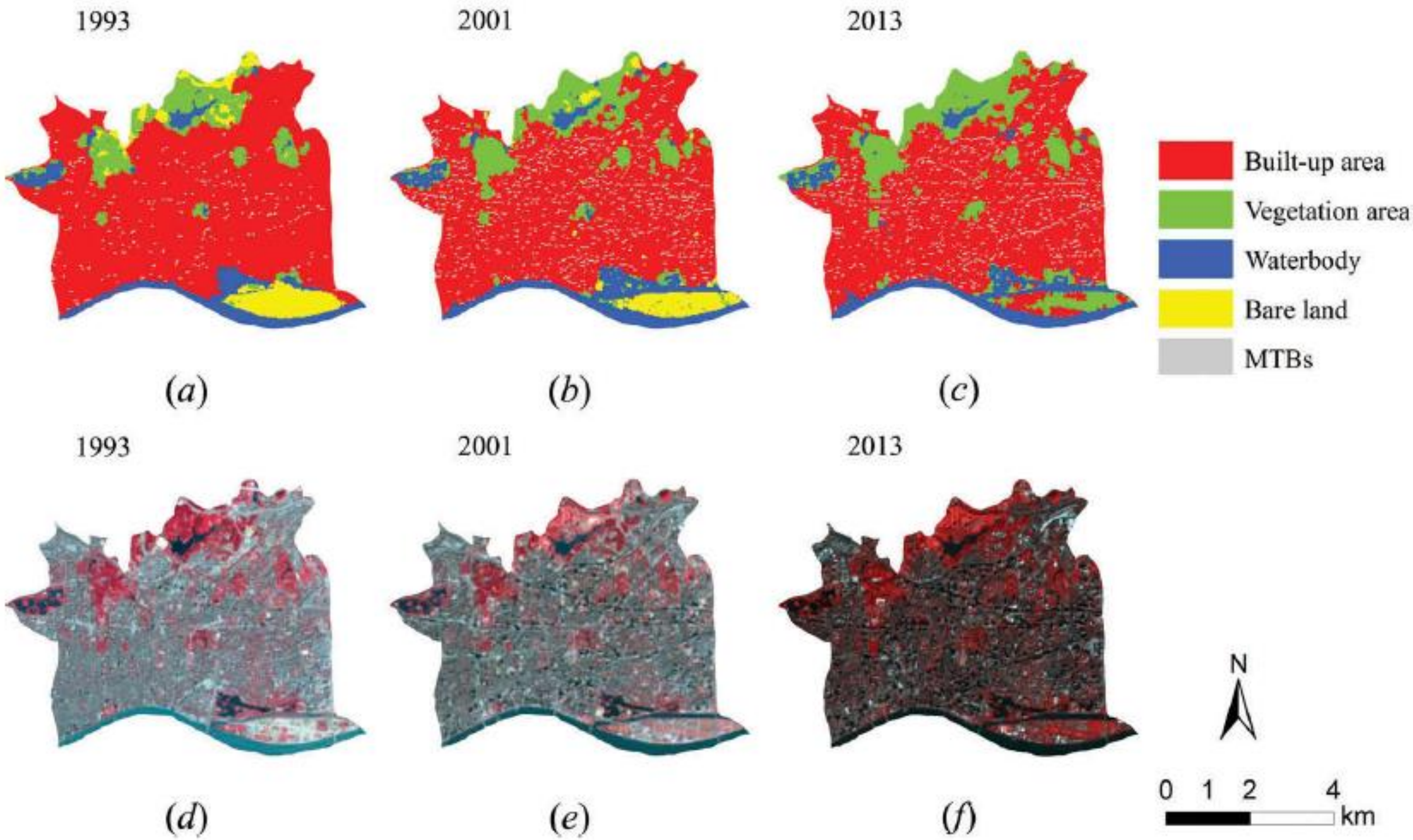


Figure 9. Land-cover maps of Yuexiu district with MTB areas: (*a*) in 1993, (*b*) in 2001, and (*c*) in 2013; and corresponding Landsat images (R: near-infrared, G: red, B: green) of Yuexiu district: (*d*) in 1993, (*e*) in 2001, and (*f*) in 2013.

Others?

- Think about where you may use the transiogram in your research and how.
- Also think about where you may use the MCRF model.

Thanks !